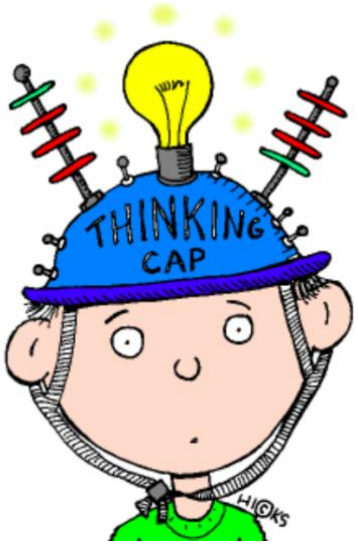
Faculty of Mathematical Applications

Skills Required for National Level Mathematics



Introduction

Welcome to National Level Mathematics.

To be successful with National Maths, at any level (3 – 5), you need to have a good understanding of the four main stages to answering mathematical questions.

These stages are:

- **Interpret the question**
- **Select a strategy**
- **Process the strategy**
- **Communicate the answer**

If you are unsure what this means, let's explain:

- **Interpret the question** - reading the question carefully and understanding what it is asking you to do.
- **Select a strategy** - decide which method will help you to find the answer.
- **Process the strategy** - apply the skills you have learned in class to solve the problem.
- **Communicate the answer** - write down your final answer ensuring you have fully answered the question.

Don't worry if you feel that you can't do this. This booklet will help you.

Interpret the Question

To enable you to interpret a mathematical question you must read the question very carefully (**more than once**) and look for key words, phrases and diagrams to help you make sense of what you are being asked to do. You need to extract the important pieces of information to use in your working.

Example

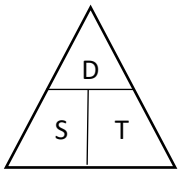
A coach left Aberdeen at 1215 and arrived at its destination at 1551. If the coach travelled at an average speed of 55mph calculated how far it travelled.

Step 1:

Look for key words – average speed, how far (distance).

Step 2:

Speed, distance, time question → remember triangle to help with formula.



Extract important information from question:

Speed – 55mph

Time – 1215 to 1551

Distance - ?

Step 3:

Choose correct formula and calculate time taken for journey.

Time – 2 hours, 36 mins

Change time into a decimal→ 36mins ÷ 60 = 0·6 hours

Time = 2·6 hours

$D = S \times T$

$D = 55 \times 2{\cdot}6$

$D = 143$

Step 4 :

D = 143 miles

Underline final answer and remember to write units.

Select a Strategy

You need to work out which topic the question relates to and decide if there is a formula you need to use to find the answer. Remember to use your formula sheet to help you where appropriate.

Remember that some problems can be solved using different strategies and you must use the one which works for you. This will become easier if you practise.

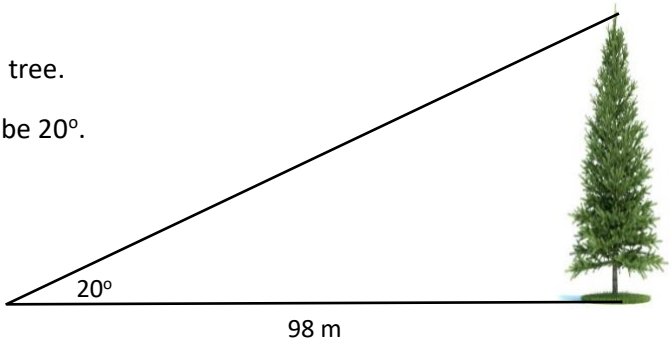
Example

Peter stands a distance of 98 m from the base of a tree.

He measures the angle of elevation and finds it to be 20°.

How high is the tree?

Give your answer to 1 decimal place.



Step 1:

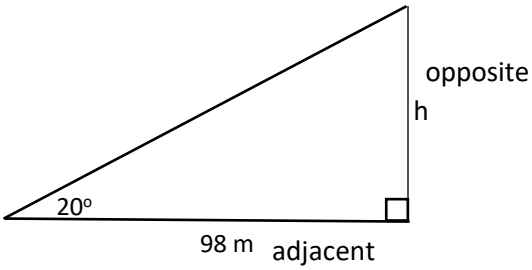
Recognise right-angled triangle and the fact that the calculation will involve 2 sides (1 missing) and 1 angle.

Step 2:

Right angled triangle question → Pythagoras or trigonometry? → choose trigonometry because the calculation involves an angle.

Step 3:

Label triangle, choose ratio and calculate h.



✓ ✓ ✓✓
SOH CAH TOA

$$\text{Tan } x^\circ = \frac{\text{Opp}}{\text{Adj}}$$

$$\text{Tan } 20^\circ = \frac{h}{98}$$

$98 \times \text{Tan } 20^\circ = h$

$h = 35{\cdot}669$

Step 4:

$\underline{h = 35{\cdot}7 \text{ m}}$

Round your answer to the required 1 decimal place, write units and underline your answer.
Remember to write your unrounded answer first before rounding.

Process the Strategy

In order to process the strategy, you need to apply all the skills you have learned in class to solve the problem – in other words – work out the answer. It is essential that you learn to show all steps in your working and check carefully for mistakes at the end.

Many pupils experience difficulties with processing as their non-calculator skills are poor. Practising times tables will help you to do many calculations. You should attempt to answer questions without a calculator as much as possible.

To allow you to gain confidence you need to practise questions and ask for help when you are unsure. If you do this on a regular basis throughout the year you will feel much more confident when sitting tests or exams.

Example

A plumber uses this formula to calculate the charges for carrying out work. He charges a call out fee plus a charge for every hour the work takes.

$C = 15n + 27$ where C = cost (£) and n = number of hours

How many hours did a job take if the total cost was £117?

Step 1:

Recognise that you have to use the given formula to work out the number of hours.

Step 2:

Understand that the given formula will need to be rearranged to calculate the required information.

Step 3:

Write down the formula, substitute the given information and begin to rearrange to find the number of hours.

$C = 15n + 27$

$117 = 15n + 27$ substitute

$15n + 27 = 117$ flip equation to put unknown term on left hand side

$15n = 117 - 27$ begin to rearrange

$15n = 90$

$n = 90 \div 15$

$n = 6$

Step 4:

$\underline{n = 6 \text{ hours}}$

Underline final answer and remember to write units.

There are two methods you can use to rearrange an equation:

- Remember when you rearrange an equation to 'change the subject of the formula' you must change the operation of each term you move from one side of the equation to the other → change side, change sign.
- If you usually balance an equation remember to do the same to both sides.

Communicate the Answer

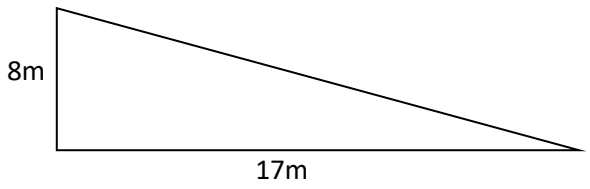
Although this might seem like the easiest part of the process, you would be surprised how easy it is to lose marks by not communicating your answer correctly.

Firstly, you must make it clear which part is your final answer – underline your answer and include units. Check if the question has asked you to round to a specific number of decimal places or significant numbers. Remember to write the unrounded answer as well as your rounded answer.

Some questions require you to write a sentence to answer the question. If this is the case make sure that you word it correctly and include a numerical comparison where appropriate.

Example

A skateboard ramp has been designed to have the dimensions shown in the diagram.



Safety regulations state that the gradient of the ramp should **not exceed 0·5**.

Does this ramp meet safety regulations? **You must show working and give a reason for your answer.**

Step 1:

Look for key words - gradient, not exceed 0·5 (not more than 0·5).

Step 2:

Use formula for gradient (from formula sheet if Nat 4).

Step 3:

Gradient = $\frac{\text{Vertical height}}{\text{Horizontal distance}}$

Gradient = $\frac{8}{17}$

Gradient = 0·47

Step 4:

Yes, the ramp does meet safety regulations as the gradient of the ramp is 0·47 and

0·47 is less than 0·5 (0·47 < 0·5).

It is very important that the final answer is worded properly and a numerical comparison is given.

Worked Examples

National 3

Stephanie wants to go on a 7 night holiday to Turkey.
The holiday will cost £295·00.
Stephanie needs to pay for her holiday in 4 months.
How much does she need to save each month?

Step 1:

Read the question very carefully.

Look for important information – cost of holiday, 4 months.

Step 2:

Write down information provided and think of a way to work out cost per month.

Overall cost - £295·00

Length of time to save – 4 months

Amount to save each month - ?

Step 3:

Division sum to work out amount per month.

£295·00 ÷ 4 = 73·75

Step 4:

Communicate final answer with units:

Stephanie will need to save £73·75 every month to pay for her holiday.

National 4

Carol is going on holiday to Japan with her parents. She takes £75 of her savings to the bank to change into yen. Exchange rate £1 = 150 ¥

(a) How many yen does she receive?

(b) At the end of her holiday Carol has 1420 yen left. How many yen did she spend?

Step 1:

Read the question carefully, look for important information/words – exchange rate.

Step 2:

Recognise this is a foreign exchange question and remember to multiply to exchange from £ sterling to another currency.

Extract important information from question:

Exchange rate £1 = 150 ¥

£75 savings

1420 yen left at end of holiday

Step 3:

a) Change £ 75 into yen:

$75 \times 150 = 11250$

b) Calculate amount of yen spent:

$11250 - 1420 = 9830$

Step 4:

Communicate final answer and remember to write units:

a) 11250 ¥

b) Carol spent 9830 ¥ on holiday.

National 5

Jack called his internet provider on six occasions to report connection problems.

On each occasion he noted the time he had to wait before speaking to an advisor.

The times (in minutes) were as follows:

13 16 10 22 5 12

a) Calculate the mean and standard deviation of these times.

b) Sophie also called the same internet provider, on several occasions, to report connection problems.

Her mean waiting time was 15 minutes and the standard deviation was 4·3 minutes.

Make two valid comments comparing Sophie's waiting times with Jack's waiting times.

Step 1:

Question tells you which topic this is – standard deviation.

Step 2:

Remember to use formulae sheet for standard deviation formula.

Part (a)

Step 3:

Calculate mean then set up table to help calculate standard deviation.

mean ($\bar{x}$) = 78 ÷ 6

$\bar{x}$ = 13

x	$x - \bar{x}$	$(x - \bar{x})^2$
13	13 – 13 = 0	0
16	16 – 13 = 3	9
10	10 – 13 = -3	9
22	22 – 13 = 9	81
5	5 – 13 = -8	64
12	12 – 13 = -1	1
		$\sum(x - \bar{x})^2$ = 164

Calculate standard deviation:

$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

$s = \sqrt{\frac{164}{6-1}}$

$s = \sqrt{\frac{164}{5}}$

s = 5·727… (remember to write unrounded answer first)

Step 4:

<u>s = 5·7</u> (to 1 d.p.)

<u>Part (b)</u>

Step 3:

Look at mean and standard deviation for both Sophie and Jack and note which is higher/lower.

Step 4:

To compare Sophie and Jack's mean:

On average Sophie's waiting times were longer than Jack's.

(Or: On average Jack's waiting times were shorter than Sophie's.)

Be sure to use the words 'on average' rather than 'mean' and relate the answer to the context of the question.

To compare standard deviation:

Sophie's waiting times were more consistent than Jack's.

(Other acceptable answers are:

Jack's waiting times were less consistent than Sophie's.

Sophie's waiting times were less spread out than Jack's.

Jack's waiting times were more spread out than Sophie's.)

The wording of this answer is very important – be careful.

Glossary

Word	Meaning
Adjacent	Adjacent sides are next to each other and are joined by a common vertex.
Bisect	To divide an angle, shape or line exactly in half.
Congruent	If two shapes are said to be congruent then they are identical in shape and size.
Deductions	The amount withheld by an employer from employee's earnings including income tax, national insurance, pension etc.
Expand	To multiply out brackets in an expression. Eg. $2(3x + 7) = 6x + 14$.
Evaluate	Find the value of … (work out the answer).
Factorise	To put an expression into brackets.
Gross Pay	Total income including basic pay, overtime, bonus, commission etc. before any deductions are made.
Interquartile Range	The difference between the lower and upper quartile. $IQR = Q_3 - Q_1$.
Justify	Another word for explain. 'Justify your answer.'
Mean	A type of average found by adding up all values and dividing by the number of values.
Median	Another type of average found by ordering the values from smallest to largest and finding the one in the middle.
Mode / Modal	Another type of average – the most common value. If two values are tied then there are two modes. If more than two values are tied there is no mode.
Net Pay	The remaining amount of an employee's gross pay after deductions are made.
Perimeter	Total distance around the outside of a shape.
Perpendicular	Two or more lines which meet at right angles.
Range	The largest value subtract the smallest value in a set of data.
Rational	A decimal number which ends or is recurring.
Semi-Interquartile Range	Half of the difference between the upper and lower quartile. $SIQR = (Q_3 - Q_1)/2$.
Simplify	To write a sum, ratio or expression in its lowest terms.
Surface Area	The total area of all sides on a 3D shape.
Tangent	A straight line that touches a curve or circle at only one point.

Helpful Websites

There are several websites which you can access for tips, notes, examples and questions to practise:

http://maths.lanark.s-lanark.sch.uk/ Password: LGSmaths

http://mathsrevision.com/

https://www.bbc.com/education

http://www.sqa.org.uk/pastpapers/findpastpaper.htm (for National 5 pupils)

http://www.national5maths.co.uk/