National
Qualifications
2017

2017 Mathematics

Advanced Higher

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The illustrative scheme covers methods which are commonly seen throughout the marking. The generic scheme indicates the rationale for which each mark is awarded. In general, markers should use the illustrative scheme and only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the detailed marking instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6 \times 6 = 12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

Comment	Working
This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x=1$

Comment	Working
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(k) **Horizontal/vertical marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark. Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ write down binomial expansion [1,3,4] •² resolve signs [3,4] •³ simplify coefficients or powers of y [2,4] •⁴ complete simplification and obtain expression [2,4,5,6]	•¹ $=\binom{3}{0}\left(\frac{2}{y^2}\right)^3+\binom{3}{1}\left(\frac{2}{y^2}\right)^2(-5y)+\binom{3}{2}\left(\frac{2}{y^2}\right)(-5y)^2+\binom{3}{3}(-5y)^3$ •^{2,3,4} $\frac{8}{y^6}-\frac{60}{y^3}+150-125y^3$	4

Notes:

1. Accept any correct form for binomial coefficients.
2. Accept negative powers of y.
3. For candidates who expand $\left(\frac{2}{y^2}+5y\right)^3$ using the Binomial Theorem •¹ and •² are not available.
4. Candidates who expand $\left(\frac{2}{y^2}-5y\right)^3$ without using the Binomial Theorem may be awarded •², •³ and •⁴ but •¹ is not available.
5. •⁴ is not available for a final expression which contains the term '$150y^0$'.
6. Do not award •⁴ where the candidate produces incorrect working subsequent to a correct simplification.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] state expression	•[1] $\frac{x^2-6x+20}{(x+1)(x-2)^2}=\frac{A}{(x+1)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}$	4
			•[2] form equation	•[2] $x^2-6x+20=A(x-2)^2+B(x+1)(x-2)+C(x+1)$	
			•[3] obtain two of A, B and C	•[3] $A=3,\ B=-2,\ C=4$	
			•[4] obtain final constant and state expression 1	•[4] $\frac{3}{(x+1)}-\frac{2}{(x-2)}+\frac{4}{(x-2)^2}$	

Notes:

1. At •[4] accept $\frac{3}{(x+1)}+\frac{-2}{(x-2)}+\frac{4}{(x-2)^2}$ but do not accept $\frac{3}{(x+1)}+-\frac{2}{(x-2)}+\frac{4}{(x-2)^2}$.

Commonly Observed Responses:

Alternative Method

$\frac{x^2-6x+20}{(x+1)(x-2)^2}=\frac{A}{x+1}+\frac{Bx+C}{(x-2)^2}$ award •[1]

$x^2-6x+20=A(x-2)^2+(Bx+C)(x+1)$ and one of $A,\ B,\ C$ award •[2]

Remaining two of $A,\ B,\ C$ ($A=3,\ B=-2,\ C=8$) award •[3]

$\frac{3}{x+1}+\frac{8-2x}{(x-2)^2}=\frac{3}{x+1}+\frac{4-2(x-2)}{(x-2)^2}$

$=\frac{3}{x+1}-\frac{2}{(x-2)}+\frac{4}{(x-2)^2}$ award •[4]

Incorrect Method

$\frac{x^2-6x+20}{(x+1)(x-2)^2}=\frac{A}{x+1}+\frac{B}{(x-2)^2}$ do not award •[1]

$x^2-6x+20=A(x-2)^2+B(x+1)$ do not award •[2]

$B=4,\ A=3$ or $B=4,\ A=1$ award •[3]

$\frac{3}{x+1}+\frac{4}{(x-2)^2}$ or $\frac{1}{x+1}+\frac{4}{(x-2)^2}$ do not award •[4]

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ evidence use of quotient rule with one term of numerator correct •² complete differentiation •³ simplify [1,2,3]	•¹ $2xe^{x^2-1}\left(x^2-1\right)-...$ •² $\dfrac{...2xe^{x^2-1}}{\left(x^2-1\right)^2}$ •³ $\dfrac{2xe^{x^2-1}\left(x^2-2\right)}{\left(x^2-1\right)^2}$	3

Notes:

1. At •³ accept $\dfrac{2x^3e^{x^2-1}-4xe^{x^2-1}}{\left(x^2-1\right)^2}$ but not accept $\dfrac{2xe^{x^2-1}\left(\left(x^2-1\right)-1\right)}{\left(x^2-1\right)^2}$ (GM Principle (l)).
2. Do not award •³ where the candidate produces further incorrect simplification subsequent to a correct answer.
3. Where a candidate differentiates incorrectly •³ may be available provided like terms are collected in the numerator. Where this is not possible the expression should be fully factorised (this need not extend to exponential functions of differing powers). Where no simplification is possible •³ is not available.

Commonly Observed Responses:

Alternative Method 1 (Product Rule)

•¹ $2xe^{x^2-1}\left(x^2-1\right)^{-1}+\ldots$

•² $\ldots-2xe^{x^2-1}\left(x^2-1\right)^{-2}$ Award •³ as per illustrative scheme.

Alternative Method 2 (Logarithmic differentiation)

•¹ $\ln|y|=\ln\left(e^{x^2-1}\right)-\ln\left|x^2-1\right|$ (modulus signs not required)

•² $\dfrac{1}{y}\dfrac{dy}{dx}=2x-\dfrac{2x}{x^2-1}$

•³ $\dfrac{dy}{dx}=\dfrac{e^{x^2-1}}{x^2-1}\left(2x-\dfrac{2x}{x^2-1}\right)$ (no further simplification required but refer to Note 2)

Alternative Method 3 (Substitution)

•¹ $u=x^2-1,\ f\left(u\right)=\dfrac{e^u}{u}$

$f'(x)=\dfrac{df}{du}\times\dfrac{du}{dx}$

•² $=\dfrac{ue^u-e^u}{u^2}\times 2x$ Award •³ as per illustrative scheme.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] evidence use of valid strategy •[2] obtain values of a and d [1]	•[1] eg $a+4d=-6$ $a+11d=-34$ •[2] $a=10,\ d=-4$	**2**

Notes:

1. Candidates who state correct values for both a and d without working may be awarded •[1] and •[2].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] set up equation •[4] rearrange to standard form [1] •[5] determine the value of n [2]	•[3] $\frac{n}{2}\left[20-4(n-1)\right]=-144$ •[4] $2n^2-12n-144=0$ •[5] $n>0\ \therefore\ n=12$	**3**

Notes:

1. •[4] may be awarded only where a quadratic equation has been expressed in standard form.
2. •[5] may be awarded only where an invalid solution for n has been discarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)	(i)	•[1] set up augmented matrix •[2] obtain two zeros [1] •[3] complete row operations [1] •[4] obtain expression for z [2,3]	•[1] $\left(\begin{array}{ccc\|c} 1 & 2 & -1 & -3 \\ 4 & -2 & 3 & 11 \\ 3 & 1 & 2\lambda & 8 \end{array}\right)$ •[2] $\left(\begin{array}{ccc\|c} 1 & 2 & -1 & -3 \\ 0 & -10 & 7 & 23 \\ 0 & -5 & 2\lambda+3 & 17 \end{array}\right)$ •[3] $\left(\begin{array}{ccc\|c} 1 & 2 & -1 & -3 \\ 0 & -10 & 7 & 23 \\ 0 & 0 & 4\lambda-1 & 11 \end{array}\right)$ •[4] $z=\dfrac{11}{4\lambda-1}$	**4**
		(ii)	•[5] state value of λ	•[5] $\lambda=\dfrac{1}{4}$	**1**
	(b)		•[6] find solution [4]	•[6] $z=-1,\ y=-3,\ x=2$	**1**

Notes:

1. Only Gaussian Elimination (i.e. a systematic approach using EROs) is acceptable for the award of •[2] and •[3].
2. Do not accept an answer of $(4\lambda-1)z=11$ when awarding •[4].
3. At •[4] accept an unsimplified expression for z eg $z=\dfrac{5\cdot5}{2\lambda-\frac{1}{2}}$.
4. Where decimal approximations are used •[6] is available only where candidates work to 3sf or better.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			•1 differentiate $5x^2$	•1 $\frac{du}{dx}=10x$ or $du=10xdx$	6
			•2 find limits for u 3	•2 $u=0, u=\frac{1}{2}$	
			•3 replace '$x\,dx$' 1,2	•3 $\frac{1}{10}\int \ldots du$	
			•4 obtain integrand 1,2	•4 $\frac{1}{10}\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}}du$	
			•5 integrate 2,3,4,5	•5 $\frac{1}{10}\left[\sin^{-1}u\right]_0^{\frac{1}{2}}$	
			•6 evaluate 2,6,7,8	•6 $\frac{\pi}{60}$	

Notes:

1. At •3 and •4 treat as bad form situations where candidates either omit limits or retain limits for x.
2. Where candidates attempt to integrate an expression containing both u and x, where x is either inside the integrand or erroneously taken outside as a constant, only •1 and •2 may be available.
3. Where candidates do not change limits but who produce working leading to $\frac{1}{10}\left[\sin^{-1}\left(5x^2\right)\right]_0^{\frac{1}{\sqrt{10}}}$, •2 may be awarded.
4. Where candidates show no working but write down $\frac{1}{10}\left[\sin^{-1}\left(5x^2\right)\right]_0^{\frac{1}{\sqrt{10}}}$, •1 is not available.
5. •5 and •6 are unavailable to candidates who having been awarded •4 subsequently proceed to $\frac{1}{10}\left[\frac{\left(1-u^2\right)^{\frac{1}{2}}}{-\frac{1}{2}\times 2u}\right]$.
6. For candidates who integrate incorrectly, •6 may be available provided division by zero does not occur.
7. For candidates who, upon integrating, obtain a trigonometric expression and then work in degrees •6 is unavailable.
8. Disregard the appearance of a decimal approximation subsequent to a simplified exact value.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)	(i)	•[1] determine value of x	•[1] $x=8$	**1**
		(ii)	•[2] find inverse [1]	•[2] $P^{-1}=\frac{1}{2}\begin{pmatrix}-1 & -2\\ 5 & 8\end{pmatrix}$	**1**
		(iii)	•[3] state transpose •[4] obtain product [2,3]	•[3] $Q'=\begin{pmatrix}2 & 4\\ -3 & y\end{pmatrix}$ •[4] $P^{-1}Q'=\begin{pmatrix}2 & -2-y\\ -7 & 10+4y\end{pmatrix}$	**2**

Notes:

1. At •[2] accept $P^{-1}=\frac{1}{2}\begin{pmatrix}-1 & -2\\ 5 & x\end{pmatrix}$.
2. For •[4] accept $P^{-1}Q'=\frac{1}{2}\begin{pmatrix}4 & -4-2y\\ -14 & 20+8y\end{pmatrix}$ but not $P^{-1}Q'=\frac{1}{2}\begin{pmatrix}-2+6 & -4-2y\\ 10-24 & 20+8y\end{pmatrix}$.
3. •[4] may be awarded only where y is present.

Commonly Observed Responses:

	(b)		•[5] state condition for singularity [1,2] •[6] obtain value for z [2]	•[5] $\det R=0$ or one row is a multiple of the other •[6] $z=15$	**2**

Notes:

1. $\det R=0$ may be stated or implied in the working for •[6].
2. For an answer of $z=15$ without justification, •[5] is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] start process •[2] obtain remainder of 29 [1] •[3] express gcd in terms of 377 and 1218 •[4] state values of a and b [2]	•[1] $1595 = 1\times 1218 + 377$ $1218 = 3\times 377 + 87$ •[2] $377 = 4\times 87 + 29$ $87 = 3\times 29 + 0$ •[3] $29 = 377 - 4(1218 - 3\times 377)$ •[4] $a = 13, b = -17$	4

Notes:

1. At •[2] the gcd does not need to be explicitly stated.
2. The minimum requirement for •[4] is $1595\times 13 + 1218\times(-17) = 29$.
3.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			•1 separate variables and write down integral equation 1,7	•1 $\int \frac{dy}{1+y^2} = \int e^{2x}\,dx$	5
			•2 integrate LHS 2	•2 $\tan^{-1} y$	
			•3 integrate RHS 3	•3 $\frac{1}{2}e^{2x}+c$	
			•4 evaluate constant of integration 2,3,4,5	•4 $c=\frac{\pi}{4}-\frac{1}{2}$	
			•5 express y in terms of x 3,5,6	•5 $y=\tan\left(\frac{1}{2}e^{2x}+\frac{\pi}{4}-\frac{1}{2}\right)$	

Notes:

1. Do not withhold •1 where dy and dx have been omitted.
2. For candidates who integrate the LHS and obtain a logarithmic expression, •2 and •4 are not available.
3. For candidates who omit a constant of integration, •3 may be awarded but •4 and •5 are unavailable.
4. At •4 accept a decimal value for the constant of integration correct to at least 3sf $(0\cdot285)$.
5. For candidates who work in degrees, •4 is unavailable but •5 may be awarded.
6. At •5 do not accept e.g. $y=\tan\left(\frac{1}{2}e^{2x}\right)+\frac{\pi}{4}-\frac{1}{2}$, $y=\tan\frac{1}{2}e^{2x}+\frac{\pi}{4}-\frac{1}{2}$.
7. Candidates who use either Integration by Parts or the Integrating Factor Method receive 0/5.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•¹ substitute formulae	•¹ $\sum_{r=1}^{n}\left(r^2+\frac{1}{3}r\right)=\frac{n(n+1)(2n+1)}{6}+\frac{1}{3}\left(\frac{n(n+1)}{2}\right)$ $=\frac{n(n+1)((2n+1)+1)}{6}$	**2**
			•² factorise fully [1]	•² $=\frac{n(n+1)^2}{3}$	

Notes:

1. At •² do not accept $\frac{n(n+1)(n+1)}{3}$ or $\frac{2n(n+1)^2}{6}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ substitute $2p$ and 9	•³ $\frac{2p(2p+1)^2}{3}$ and $\frac{9(9+1)^2}{3}$ $\frac{2p(2p+1)^2}{3}-\frac{9(9+1)^2}{3}$	**2**
			•⁴ obtain expression	•⁴ $=\frac{2p(2p+1)^2}{3}-300$	

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.			**Method 1**		5
			•1 take logarithms of both sides and apply rule 1	•1 $\ln y = \left(2x^3+1\right)\ln x$	
			•2 differentiate LHS	•2 $\frac{1}{y}\frac{dy}{dx}$	
			•3 evidence use of product rule and one term correct 2	•3 $6x^2 \ln x$ or $\frac{2x^3+1}{x}$	
			•4 complete differentiation 2	•4 $6x^2 \ln x + \frac{2x^3+1}{x}$	
			•5 write $\frac{dy}{dx}$ in terms of x	•5 $\frac{dy}{dx} = x^{2x^3+1}\left(6x^2 \ln x + \frac{2x^3+1}{x}\right)$	

Notes:

1. Accept 'log' in lieu of 'ln'.
2. For candidates who do not attempt to use the product rule, •3 and •4 are not available.

Commonly Observed Responses:

For candidates who express y as $e^{\left(2x^3+1\right)\ln x}$ marks may be awarded as follows:

•1 is not available

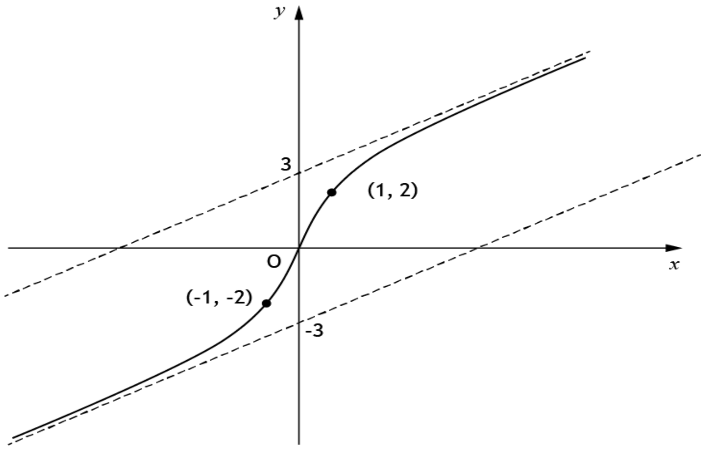
•2 writing in the form $y = e^{\left(2x^3+1\right)\ln x}$

•3 apply chain rule: $\frac{dy}{dx} = e^{\left(2x^3+1\right)\ln x} \times \frac{d}{dx}\left(\left(2x^3+1\right)\ln x\right)$

•4 evidence use of product rule and one term correct : $6x^2 \ln x$ or $\frac{2x^3+1}{x}$

•5 complete differentiation: $\frac{dy}{dx} = x^{2x^3+1}\left(6x^2 \ln x + \frac{2x^3+1}{x}\right)$ or $e^{\left(2x^3+1\right)\ln x}\left(6x^2 \ln x + \frac{2x^3+1}{x}\right)$

Note: If a candidate writes $y = e^{\left(2x^3+1\right)} \ln x$, only marks •4 and •5 are available.

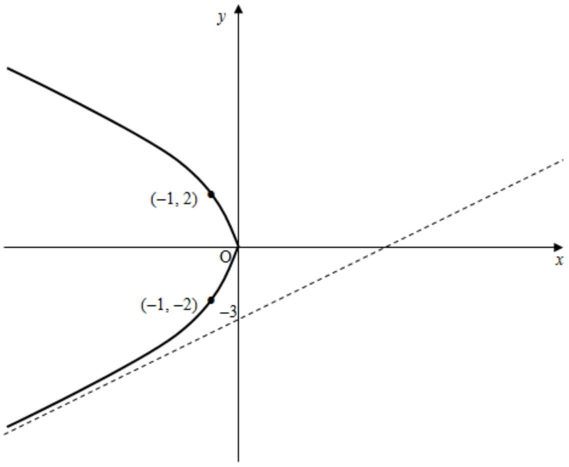
Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] show half-turn symmetry and indicate $(1,2)$ [1,2] •[2] demonstrate graph approaching parallel asymptote through $(0,3)$ [3,4]	•[1,2] 	**2**

Notes:

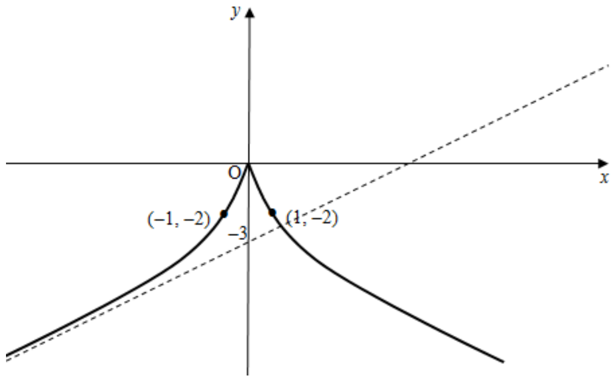
1. To award •[1] the candidate's graph should exhibit a smooth change in concavity at the origin.
2. Evidence of $(1,2)$ may appear in (b).
3. At •[2] accept $y=\frac{1}{2}x+3$ in lieu of $(0,3)$.
4. Where a candidate's graph diverges from the asymptote in quadrant 1, •[2] is not available.
5. For Graph 1 in the **Commonly Observed Responses** •[1], •[2], •[3] and •[4] are not available.
6. For Graph 2 in the **Commonly Observed Responses** •[1], •[2] and •[3] are not available but •[4] may be available where a second asymptote appears in (b).

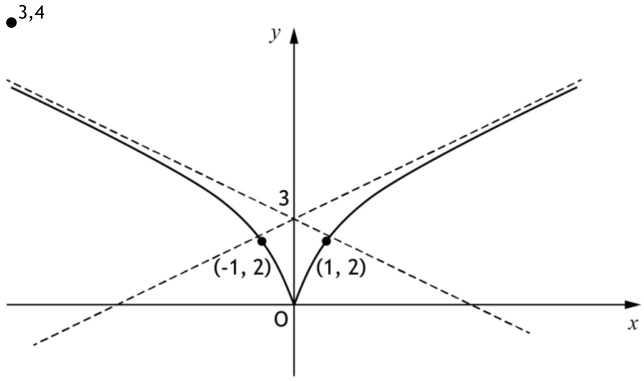
Commonly Observed Responses:

Graph 1



Graph 2 (one asymptote only)



Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)		•3 apply modulus function to graph obtained in (a) [1,4] •4 illustrate asymptotes meeting on the y-axis [1,2,3]	•3,4 	**2**

Notes:

1. To receive any credit, a candidate's graph from (a) must have a section lying in quadrant 1.
2. •4 is still available where a candidate's graph diverges from the asymptotes.
3. At •4 disregard the application of the modulus function to asymptotes.
4. Showing the image points is not required at •3.

Commonly Observed Responses:

	(c)		**State the range of values of $f'(x)$ given that $f'(0)=2$.**		
			•5 state range [1,2,3]	•5 $\frac{1}{2}< f'(x)\leq 2$	**1**

Commonly Observed Responses:

Notes:

1. Do not accept $\frac{1}{2}\leq f'(x)\leq 2$ or $\frac{1}{2}< f'(x)< 2$.
2. Accept '$f'(x)>\frac{1}{2}$ and $f'(x)\leq 2$' but not '$f'(x)>\frac{1}{2}$ or $f'(x)\leq 2$'.
3. Accept '$f'(x)$ is greater than $\frac{1}{2}$ and $f'(x)$ is less than or equal to 2'. Do not accept '$f'(x)$ is between $\frac{1}{2}$ and 2'.

Question			Generic scheme	Illustrative scheme	Max mark
13.			•[1] write down contrapositive statement [1,2,7,8]	•[1] The contrapositive of the original statement is : If n is odd then n^2 is odd	**4**
			•[2] write down appropriate form for n [3,4,7]	•[2] $n = 2k+1$, $k \in \mathbb{Z}$	
			•[3] show n^2 is odd [5,6,7]	•[3] $n^2 = 2\left(2k^2+2k\right)+1$ which is odd	
			•[4] communicate	•[4] contrapositive statement is true therefore original statement is true	

Notes:

1. A candidate who incorrectly states the contrapositive as n^2 is odd $\Rightarrow n$ is odd (or any other statement masquerading as the contrapositive) and subsequently demonstrates that when n is odd then n^2 is odd may be awarded •[3] only.
2. The minimum requirement for •[1] is a statement such as:
   n is odd $\Rightarrow$ n^2 is odd
   n is odd then n^2 is odd
   n is odd is a sufficient condition for n^2 is odd
   n is odd only if n^2 is odd
   n^2 is odd when n is odd
   Do not accept " n is odd, n^2 is odd" or " n is odd when n^2 is odd
3. At •[2] $k \in \mathbb{Z}$ is not required. Accept the form $n = 2k \pm a$, where a is a specified odd number.
4. For candidates who proceed from:
   eg $n = 2n+1$ — •[2] and •[4] are not available
   eg $n = 2k$ — •[2], •[3] and •[4] are not available
   eg $n = k+1$ — •[2], •[3] and •[4] are not available (n is not always odd)
   eg $n = 4k+1$ — •[2] is not available (not all odd numbers covered by this form)
5. At •[3] accept $n^2 = 4(\ldots)+1$, $n^2 = 2k(\ldots)+1$ or $n^2 = 4k(\ldots)+1$.
6. At •[3] candidates must state a conclusion eg "which is odd".
7. Candidates who carry out a proof by contradiction may be awarded •[2] and •[3] only.
8. Candidates who write $\neg Q \Rightarrow \neg P$ may be awarded •[1] where they either identify P and Q or have written $P \Rightarrow Q$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.			•1 construct auxiliary equation [1,9]	•1 $m^2-6m+9=0$	10
			•2 solve auxiliary equation and state CF [2,3,4,5,6,7,9]	•2 $y=Ae^{3x}+Bxe^{3x}$	
			•3 state PI	•3 $y=C\sin x+D\cos x$	
			•4 obtain first and second derivatives of PI	$\frac{dy}{dx}=C\cos x-D\sin x$ •4 $\frac{d^2y}{dx^2}=-C\sin x-D\cos x$	
			•5 substitute	•5 $-C\sin x-D\cos x$ $-6(C\cos x-D\sin x)$ $+9(C\sin x+D\cos x)=8\sin x+19\cos x$	
			•6 derive equations	$8C+6D=8$ •6 $-6C+8D=19$	
			•7 obtain both constants of PI	•7 $C=-\frac{1}{2},\ D=2$	
			•8 differentiate general solution [5,6,7,9,10]	•8 $\frac{dy}{dx}=3Ae^{3x}+Be^{3x}+3Bxe^{3x}-\frac{1}{2}\cos x-2\sin x$	
			•9 determine first constant of general solution [7,8,9]	•9 $A=5$ or $B=-14$	
			•10 determine second constant and state particular solution [3,7,9,10]	•10 $y=5e^{3x}-14xe^{3x}-\frac{1}{2}\sin x+2\cos x$	

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. $\bullet^1$ is **not** available where '$=0$' has been omitted.
2. $\bullet^2$ can be awarded if the Complementary Function appears later as part of the general solution, as opposed to being explicitly stated immediately after solving the Auxiliary Equation.
3. Do not penalise the omission of '$y=\ldots$' provided it appears at $\bullet^{10}$.
4. For candidates who obtain a CF of $y=Ae^{-3x}+Bxe^{-3x}$ only $\bullet^2$ is not available. In this case the particular solution is $y=5e^{-3x}+16xe^{-3x}-\frac{1}{2}\sin x+2\cos x$.
5. For candidates who obtain two real and distinct roots $\bullet^2$ and $\bullet^8$ are not available.
6. For candidates who obtain roots of the form $p\pm qi$: if $p=0$ and $q\neq 1$ $\bullet^2$ and $\bullet^8$ are not available, otherwise only $\bullet^2$ is not available.
7. For candidates who obtain a CF of $y=Ae^{3x}+Be^{3x}$, $\bullet^2$, $\bullet^8$, $\bullet^9$ and $\bullet^{10}$ are not available.
8. Where a candidate substitutes the given conditions into the CF to obtain values of A and B and then finds the PI correctly, $\bullet^9$ is not available.
9. Where a candidate does not find a PI only $\bullet^1$, $\bullet^2$, $\bullet^8$, $\bullet^9$ and $\bullet^{10}$ are available.
10. Where an error in the differentiation of the general solution results in the value of B being unobtainable then $\bullet^{10}$ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] obtain direction vector [1,2,4] •[2] state parametric equations [3,4,5]	•[1] $\mathbf{d}=\begin{pmatrix}2\\6\\-1\end{pmatrix}$ or multiple thereof •[2] $x=2\lambda+7$ $y=6\lambda+8$ $z=-\lambda+1$ or $x=2\lambda-3$ $y=6\lambda-22$ $z=-\lambda+6$ Or equivalent	**2**

Notes:

1. For candidates who express the equation in either symmetric or vector form •[1] is available for evidence of a correct direction vector; •[2] is unavailable unless parametric equations appear at (c).
2. Throughout the question accept horizontal vector notation eg $(2,6,-1)$.
3. A correct answer with no working receives full marks.
4. For an incorrect answer containing the correct direction vector but with no working, •[1] is available.
5. For an answer with an incorrect direction vector and no working neither •[1] nor •[2] are available.

Commonly Observed Responses:

Unsimplified direction vector: $\mathbf{d}=\begin{pmatrix}-10\\-30\\5\end{pmatrix}$.

Parametric equations: $x=-10\lambda+7$, $y=-30\lambda+8$, $z=5\lambda+1$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] identify vectors	•[3] any two from $\overrightarrow{PQ}=\begin{pmatrix}-1\\1\\-2\end{pmatrix}$, $\overrightarrow{PR}=\begin{pmatrix}-5\\6\\-8\end{pmatrix}$, $\overrightarrow{QR}=\begin{pmatrix}-4\\5\\-6\end{pmatrix}$ or equivalent	4
			•[4] evidence of strategy for finding normal [1]	•[4] $\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-1 & 1 & -2\\-5 & 6 & -8\end{vmatrix}$ or equivalent	
			•[5] calculate normal	•[5] $\mathbf{n}=\begin{pmatrix}4\\2\\-1\end{pmatrix}$	
			•[6] obtain equation	•[6] $4x+2y-z=1$	

Notes:

1. Do not award •[4] where the position vectors of P, Q or R are used.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[7] substitute into equation of plane	•[7] $4(2\lambda+7)+2(6\lambda+8)-(-\lambda+1)=1$	3
			•[8] find λ	•[8] $\lambda=-2$	
			•[9] determine coordinates of H [1]	•[9] $\mathrm{H}(3,-4,3)$	

Notes:

1. Do not accept a position vector at •[9].

Commonly Observed Responses:

For candidates who use the unsimplified direction vector from **Commonly Observed Responses** in (a) , $\lambda=\frac{2}{5}$.

Question			Generic scheme	Illustrative scheme	Max mark
16.			•1 state form of integral [1,2,3]	•1 $V=\pi\int x^2\,dy$ or $V=\pi\int\left(f\left(y\right)\right)^2\,dy$	5
			•2 rearrange and substitute for x^2	•2 $V=\pi\int\left(9-\frac{9}{4}y^2\right)dy$	
			•3 calculate limits to match variable [4]	•3 $\int_0^2 \ldots dy$ or $y=0, y=2$	
			•4 integrate	•4 $V=\pi\left[9y-\frac{3y^3}{4}\right]_0^2$	
			•5 evaluate [5,6]	•5 $V=12\pi$ (cubic units)	

Notes:

1. dy must appear for •1 to be awarded.
2. •1 may be awarded at •2.
3. For candidates who write $V=\pi\int x^2\,dx$, $V=\pi\int y^2\,dy$ or $V=\pi\int y^2\,dx$ and proceed to:

   (a) $V=\pi\int\left(9-\frac{9}{4}y^2\right)dy$ full credit may still be available

   (b) $V=\pi\int\left(4-\frac{4}{9}x^2\right)dx$ •2, •3, •4 and •5 may still be available

   (c) $\pi\left[\frac{x^3}{3}\right]$ or $\pi\left[\frac{y^3}{3}\right]$ only •3 is available
4. •3 may be awarded at •4
5. •5 is not available where a candidate's evaluation necessarily leads to a negative answer.
6. At •5 units are not required.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17	(a)		•1 state second root	•1 $2-i$	1
Notes:					
Commonly Observed Responses:					
	(b)		•2 obtain two linear factors	•2 $z-(2+i),\ z-(2-i)$	6
			•3 obtain quadratic factor	•3 z^2-4z+5	
			•4 set up algebraic division or equivalent	•4 $z^2-4z+5\,\overline{\big)\,z^4-6z^3+16z^2-22z+q}$	
			•5 complete algebraic division	•5 $\begin{array}{r} z^2-2z+3 \\ z^2-4z+5\,\overline{\big)\,z^4-6z^3+16z^2-22z+q} \\ z^4-4z^3+5z^2 \\ -2z^3+11z^2-22z+q \\ -2z^3+8z^2-10z \\ 3z^2-12z+q \\ 3z^2-12z+15 \\ q-15 \end{array}$	
			•6 state value of q [1,2]	•6 $q=15$	
			•7 obtain remaining two roots	•7 $1\pm\sqrt{2}\,i$	

Notes

1. For candidates who substitute either $2+i$ or $2-i$ into the equation, obtain a correct value of q but who do not exhibit any other working, only •6 may be awarded.
2. •6 not available for a non-integer value of q.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Alternative Method 1

•4 $z=2+i,\ z^2=3+4i,\ z^3=2+11i,\ z^4=-7+24i$ and substitute to get $q=15$

•5 $z^4-6z^3+16z^2-22z+15=\left(z^2-4z+5\right)\left(z^2+az+3\right)$ for some a

•6 $a=-2$

Alternative Method 2

•2 set up synthetic division - coefficients and known root

•3
$$\begin{array}{c|ccccc} 2+i & 1 & -6 & 16 & -22 & q \\ & & 2+i & -9-2i & 16+3i & -15 \\ \hline & 1 & -4+i & 7-2i & -6+3i & \boxed{q-15} \end{array}$$

•4 $q=15$

•5
$$\begin{array}{c|cccc} 2-i & 1 & -4+i & 7-2i & -6+3i \\ & & 2-i & -4+2i & 6-3i \\ \hline & 1 & -2 & 3 & \boxed{0} \end{array}$$

•6 z^2-2z+3 stated or implied

Alternative Method 3

•2 set up synthetic division - coefficients and known root

•3
$$\begin{array}{c|ccccc} 2-i & 1 & -6 & 16 & -22 & q \\ & & 2-i & -9+2i & 16-3i & -15 \\ \hline & 1 & -4-i & 7+2i & -6-3i & \boxed{q-15} \end{array}$$

•4 $q=15$

•5
$$\begin{array}{c|cccc} 2+i & 1 & -4-i & 7+2i & -6-3i \\ & & 2+i & -4-2i & 6+3i \\ \hline & 1 & -2 & 3 & \boxed{0} \end{array}$$

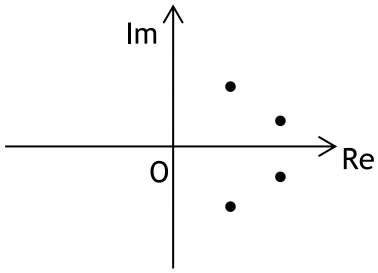
•6 z^2-2z+3 stated or implied

Alternative Method 4

•4 $z^4-6z^3+16z^2-22z+q=\left(z^2-4z+5\right)\left(az^2+bz+c\right)$

•5 $z^4-6z^3+16z^2-22z+q=az^4+z^3(b-4a)+z^2(c-4b+5a)+z(-4c+5b)+5c$ leading to $a=1,\ b=-2,\ c=3$

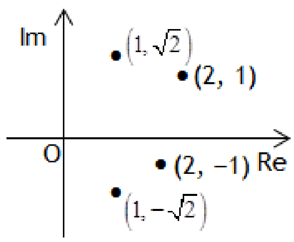
•6 $q=15$

Question			Generic scheme	Illustrative scheme	Max mark
17.	(c)		•[8] show all four solutions on an Argand diagram [1,2,3,4]	•[8] 	1

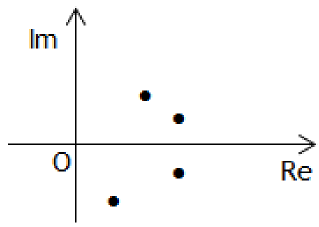
Notes:

1. Do not penalise the omission of the labels on the axes.
2. •[8] is available only where 4 roots are illustrated.
3. Positional information is required for •[8]. In the illustrative scheme this is provided by the relative positions of the points. Where points are plotted inaccurately, positional information may be provided by coordinates eg $(2,1)$ or the numbers 2 and 1 indicated on the appropriate axes. Accept $(2,i)$. The label $2+i$ is not of itself sufficient to award •[8].

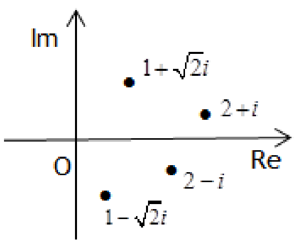
Award •[8]
Points not in correct position relative to one another but coordinates given.



Do not award •[8]
Points not in correct position relative to one another and no coordinates given.



Do not award •[8]
Points not in correct position relative to one another and no coordinates given.



4. Accept separate labelled Argand diagrams.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)		•1 evidence of use of product rule to find either $\frac{dx}{dt}$ or $\frac{dy}{dt}$ with one term correct	•1 eg $\frac{dx}{dt} = \cos t + ...$	5
			•2 obtain $\frac{dx}{dt}$ or $\frac{dy}{dt}$	•2 $\frac{dx}{dt} = \cos t - t\sin t$	
			•3 obtain remaining derivative	•3 $\frac{dy}{dt} = \sin t + t\cos t$	
			•4 state formula for instantaneous speed	•4 $\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ stated or implied at •5	
			•5 obtain expression [1,2]	•5 $\sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}$ $= \sqrt{1+t^2}$	

Notes:

1. At •5 the simplification to $\sqrt{1+t^2}$ is not required.
2. •5 may only be awarded for substitution into an expression of the form $\sqrt{(...)^2 + (...)^2}$.

Commonly Observed Responses:

	(b)		•6 evidence of valid strategy to find value of t and obtain at least one non-zero solution [1]	•6 $0 = t\sin t$ and eg $t = \pi$	2
			•7 choose correct value for t and calculate speed [1,2]	•7 $t = 3\pi$ speed = $\sqrt{1+9\pi^2}$	

Notes:

1. For candidates who obtain an expression for $\frac{dy}{dx}$ rather than instantaneous speed, •6 and •7 are still available.
2. At •7 accept a decimal answer provided it is accurate to at least 3sf $(9\cdot48)$.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]